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AEE 331 - Heat Transfer - Project I

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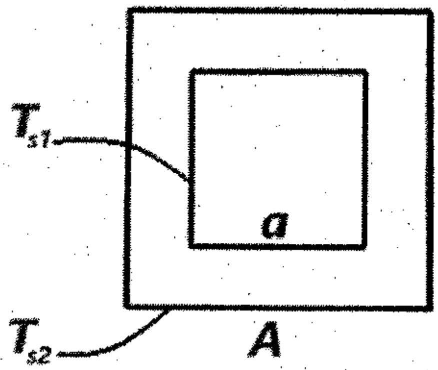


Consider the 2-D configuration in which two square are nested (a plate with a square hole in the middle). The surfaces of the squares are at constant temperature. A steady-state heat transfer occurs between these two surfaces due to the temperature difference.

a) Divide the area between these two surfaces into grids. Use a minimum of 3 grids for distance L.

b) Derive formulations via energy balance method. Make necessary simplifications. You may use any physical values for the configuration (Temperatures, dimensions etc.).

c) Solve the equations with a suitable method and obtain the temperature distributions for different A/a dimensions. Calculate the heat transfer between two surfaces per unit meter. Use these calculated values to obtain a shape factor for this geometry:

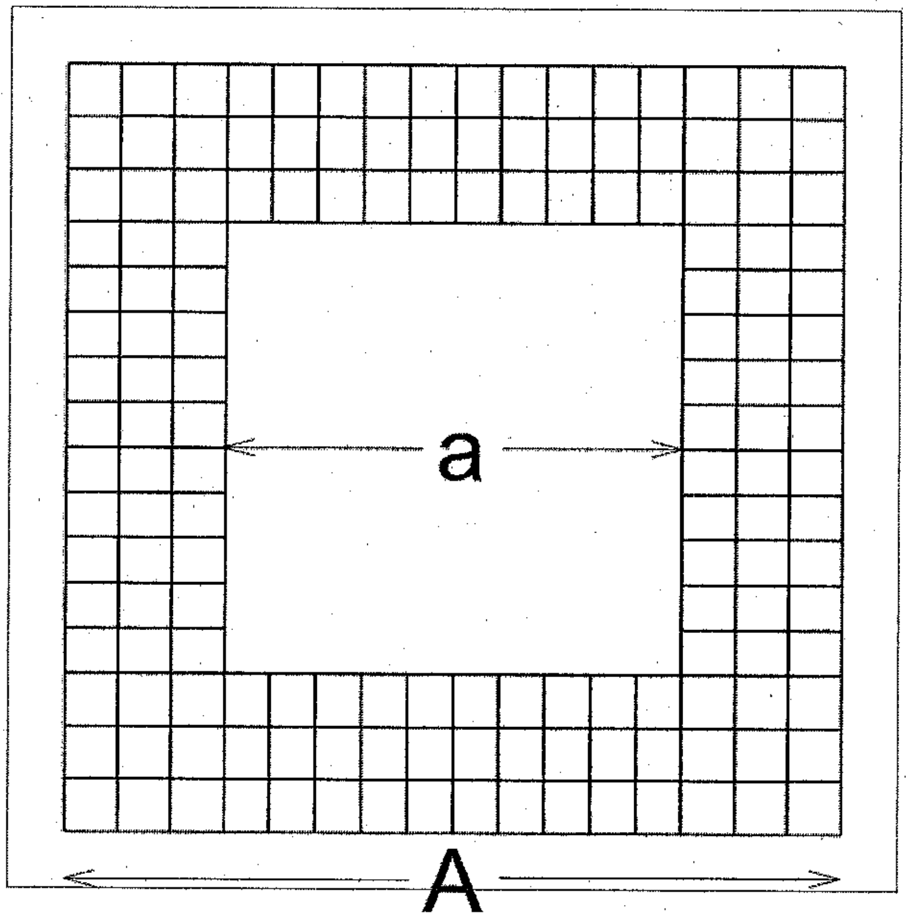
$Q=Sk\Delta T$

The shape factor should be in terms of A/a ratio. Consider these six ratios (you can consider more if you wish): 1.1, 1.2, 1.3, 1.5, 1.6, 1.7

d) Plot A/a versus S.

e) Plot also the temperature distributions for cases A/a = 1.2, 1.5

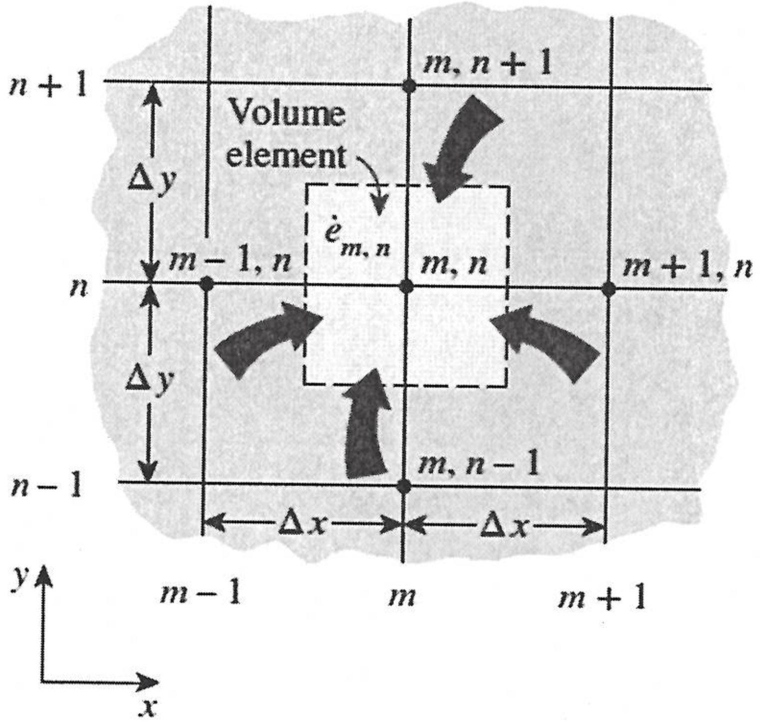
We divided the area between these two surfaces into 3 grids for distance L.



For volume element energy balance equation could be expressed as:

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{in 3D surfaces}\\ \text{of element}\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation inside}\\ \text{the element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{pmatrix}$$

$$\dot{Q}_{cond,x} + \dot{Q}_{cond,y} + \overset{\text{2D} \to 0}{\cancel{\dot{Q}_{cond,z}}} + \overset{\text{No heat gen.} \to 0}{\cancel{\dot{E}_{gen\,element}}} = \overset{\text{Steady state} \to 0}{\cancel{\frac{\Delta E_{element}}{\Delta t}}} = 0$$

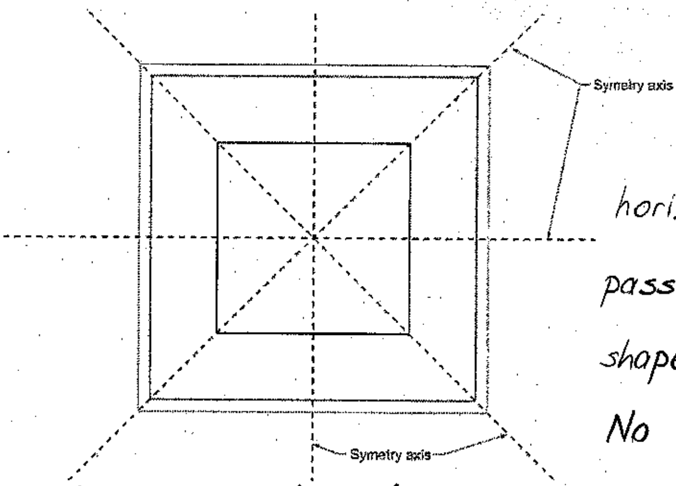


For example, for interior node (m,n) shown above energy balance equation becomes:

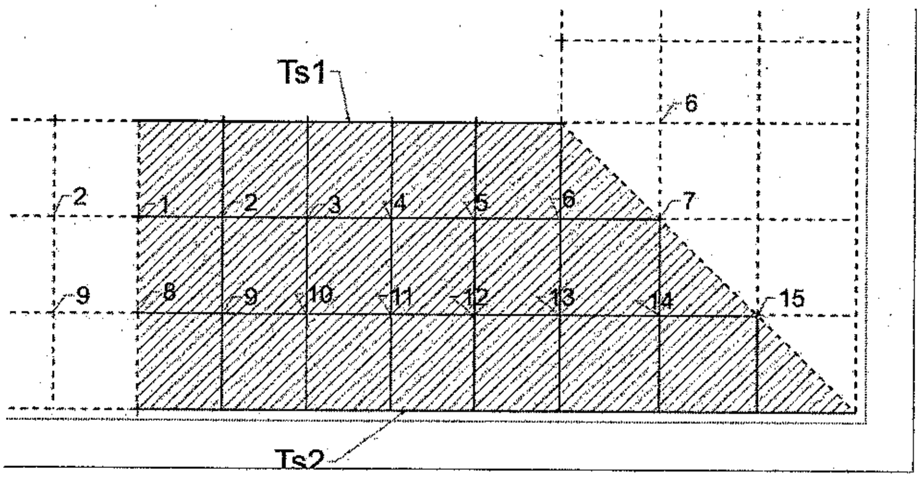
$$k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k\Delta y \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n-1} - T_{m,n}}{\Delta y} = 0$$

In the case of square meshed ($\Delta x = \Delta y$) regions equation could simplified to:

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0$$



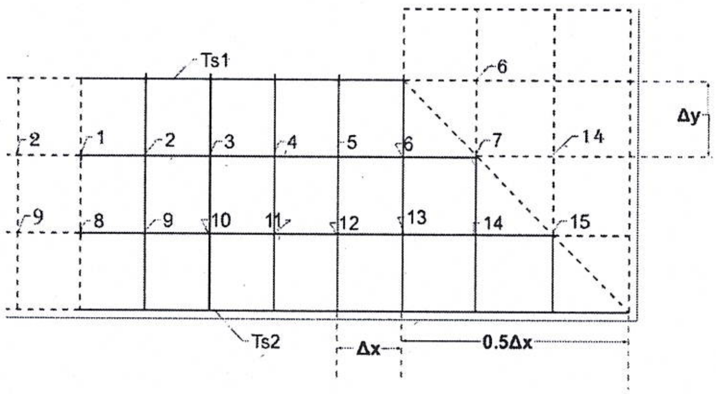
There is symmetry about the horizontal, vertical and diagonal lines passing through the midpoint of the shape as indicated on the figure. No heat can across a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite difference formulation. Then the nodes in the middle of the symmetry lines can be treated as interior nodes by using mirror images.

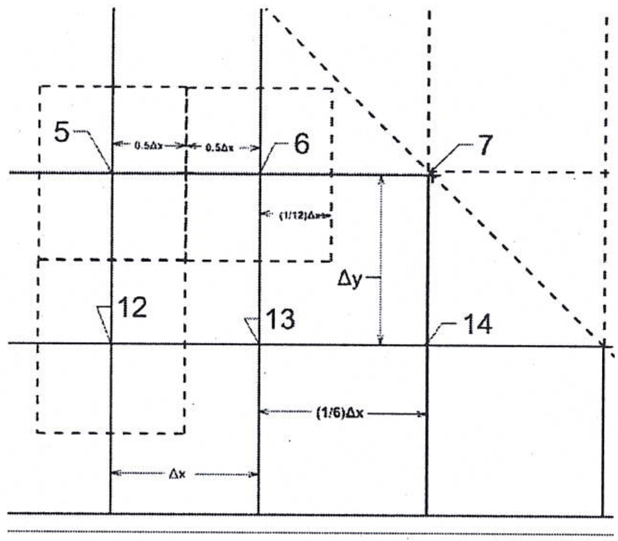


Therefore, we need to consider only one-eight of the geometry in the solution as shown above. We'll use that aproach in following steps of report.

SCALE 1 : 1

$$\frac{\Delta x}{\Delta y} = 6$$





for point 1;

$T_{s_1} + 2T_2 + T_8 - 4T_1 = 0$

for point 2;

$T_{s_1} + T_1 + T_3 + T_9 - 4T_2 = 0$

for point 3;

$T_{s_1} + T_2 + T_4 + T_{10} - 4T_3 = 0$

for point 4;

$T_{s_1} + T_3 + T_5 + T_{11} - 4T_4 = 0$

for point 5;

$T_{s_1} + T_4 + T_6 + T_{12} - 4T_5 = 0$

for point 7;

$2T_6 + 2T_{14} - 4T_7 = 0$

for point 8;

$T_{s_2} + 2T_9 + T_1 - 4T_8 = 0$

for point 9;

$T_{s_2} + T_8 + T_{10} + T_2 - 4T_9 = 0$

for point 10;

$T_{s_2} + T_9 + T_{11} + T_3 - 4T_{10} = 0$

for point 11;

$T_{s_2} + T_{10} + T_{12} + T_4 - 4T_{11} = 0$

for point 12;

$T_{s_2} + T_{11} + T_{13} + T_5 - 4T_{12} = 0$

for point 14;

$T_{s_2} + T_7 + T_{13} + T_{15} - 4T_{14} = 0$

for point 15;

$2T_{14} + 2T_{s_2} - 4T_{15} = 0$

for point 6 and 13 nodal area normal to x axis is $\left(\Delta x/2 + 1/12\, \Delta x\right)(1)$

nodal area hormal to y axis is $(\Delta y)(1)$

point 6:

$$\frac{\Delta y}{\Delta x}(T_5 - T_6) + \frac{\Delta y}{\Delta x/6}(T_7 - T_6) + \frac{7/12\,\Delta x}{\Delta y}(T_{13} - T_6) + \frac{7/12\,\Delta x}{\Delta y}(T_{s_1} - T_6) = 0$$

$$T_5 - T_6 + 6T_7 - 6T_6 + 7/3\,T_{13} - 7/3\,T_6 + 7/3\,T_{s_1} - 7/3\,T_6 = 0$$

$$T_5 + \frac{(-35)}{3}T_6 + 6T_7 + 7/3\,T_{13} + 7/3\,T_{s_1} = 0$$

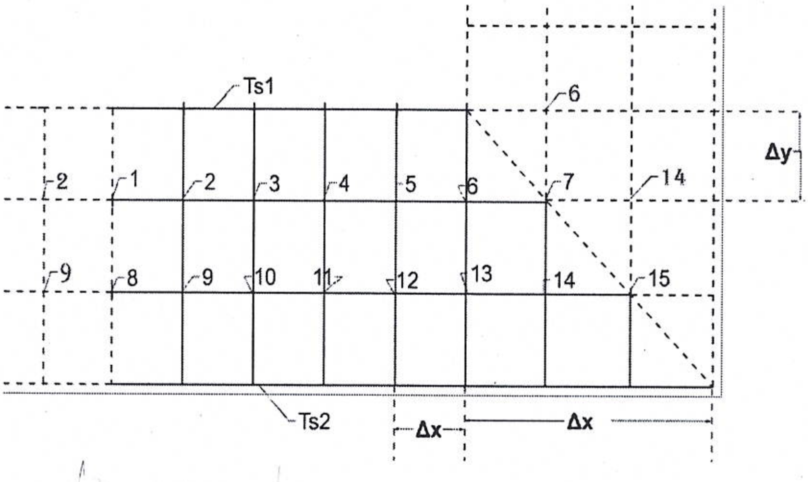
point 13:

$$\frac{\Delta y}{\Delta x}(T_{12} - T_{13}) + \frac{\Delta y}{\Delta x/6}(T_{14} - T_{13}) + \frac{7/12\,\Delta x}{\Delta y}(T_6 - T_{13}) + \frac{7/12\,\Delta x}{\Delta y}(T_{s_2} - T_{13}) = 0$$

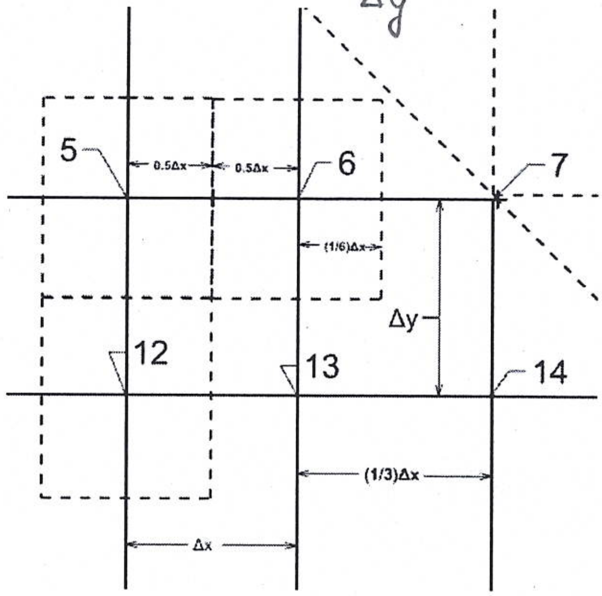
$$T_{12} - T_{13} + 6T_{14} - T_{13} + 7/3\,T_6 - 7/3\,T_{13} + 7/3\,T_{s_2} - 7/3\,T_{13} = 0$$

$$T_{12} - \frac{35}{3}T_{13} + 6T_{14} + 7/3\,T_6 + 7/3\,T_{s_2} = 0$$

SCALE 1:2



$$\frac{\Delta x}{\Delta y} = 3$$



for all scales all energy balance equations for points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15 are identical respectively.

for point 6 and 13 nodal area normal to x axis is $(\Delta x/6 + \Delta x/2)(1)$
nodal area normal to y axis is $(\Delta y)(1)$

for point 6;

$$\frac{4/6\,\Delta x}{\Delta y}(T_{s_1} - T_6) + \frac{4/6\,\Delta x}{\Delta y}(T_{13} - T_6) + \frac{\Delta y}{\Delta x}(T_5 - T_6) + \frac{\Delta y}{\Delta x/3}(T_7 - T_6) = 0$$

$$4T_{s_1} - 4T_6 + 4T_{13} - 4T_6 + T_5 - T_6 + 3T_7 - 3T_6 = 0$$

$$T_5 - 12T_6 + 3T_7 + 4T_{13} + 4T_{s_1} = 0$$

for point 13;

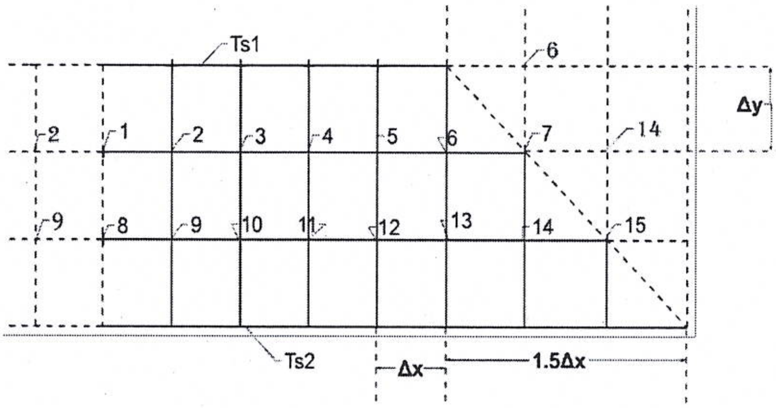
$$\frac{4/6\,\Delta x}{\Delta y}(T_6 - T_{13}) + \frac{4/6\,\Delta x}{\Delta y}(T_{s_2} - T_{13}) + \frac{\Delta y}{\Delta x}(T_{12} - T_{13}) + \frac{\Delta y}{\Delta x/3}(T_{14} - T_{13}) = 0$$

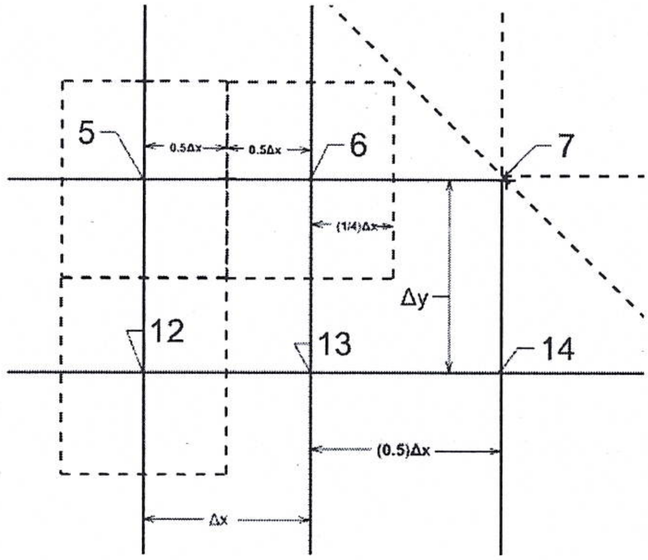
$$4T_6 - 4T_{13} + 4T_{s_2} - 4T_{13} + T_{12} - T_{13} + 3T_{14} - 3T_{13} = 0$$

$$4T_6 + T_{12} - 12T_{13} + 3T_{14} + 4T_{s_2} = 0$$

SCALE 1:3

$$\frac{\Delta x}{\Delta y} = 2$$





for all scales all energy balance equations for points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15 are identical respectively.

for point 6 and 13 nodal area normal to x axis is $(1/4\Delta x + \Delta x/2)(1)$
nodal area normal to y axis is $(\Delta y)(1)$

for point 6;

$$\frac{\Delta y}{\Delta x}(T_5 - T_6) + \frac{\Delta y}{0{,}5\Delta x}(T_7 - T_6) + \frac{3/4\,\Delta x}{\Delta y}(T_{s_1} - T_6) + \frac{3/4\,\Delta x}{\Delta y}(T_{13} - T_6) = 0$$

$$T_5 - T_6 + 2T_7 - 2T_6 + 3T_{s_1} - 3T_6 + 3T_{13} - 3T_6 = 0$$

$$T_5 - 9T_6 + 2T_7 + 3T_{s_1} + 3T_{13} = 0$$

for point 13;

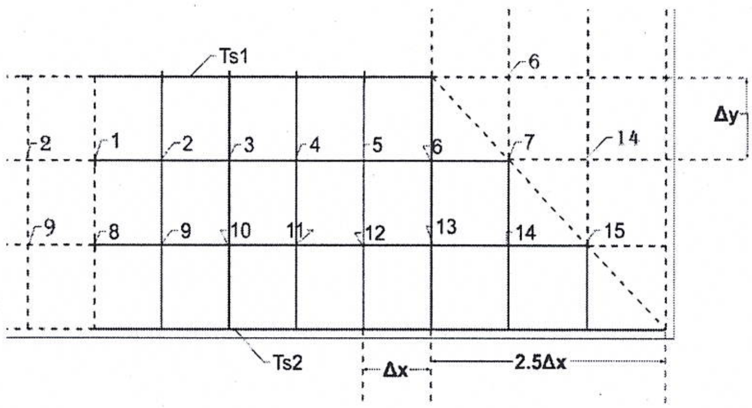
$$\frac{3/4\,\Delta x}{\Delta y}(T_6 - T_{13}) + \frac{3/4\,\Delta x}{\Delta y}(T_{s_2} - T_{13}) + \frac{\Delta y}{0{,}5\Delta x}(T_{14} - T_{13}) + \frac{\Delta y}{\Delta x}(T_{12} - T_{13}) = 0$$

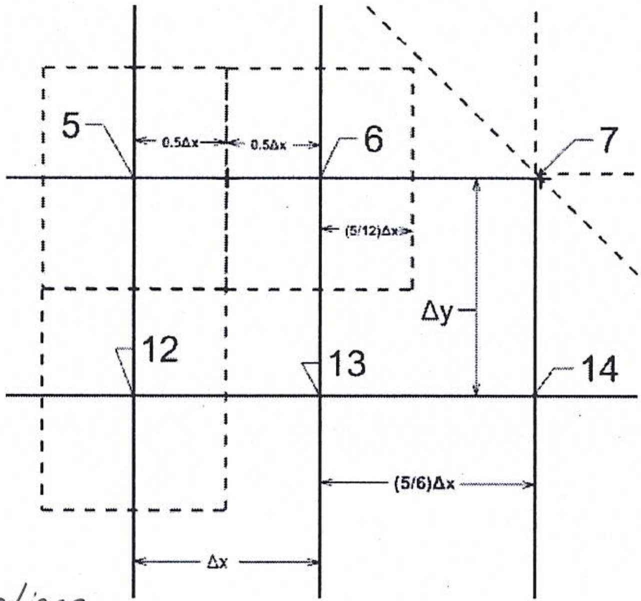
$$3T_6 - 3T_{13} + 3T_{s_2} - 3T_{13} + 2T_{14} - 2T_{13} + T_{12} - T_{13} = 0$$

$$3T_6 + 3T_{s_2} - 9T_{13} + 2T_{14} + T_{12} = 0$$

SCALE 1:5

$$\frac{\Delta x}{\Delta y} = \frac{6}{5}$$





for all scales all energy balance equations
for points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15 are identical respectively.

for points 6 and 13 nodal area normal to x axis is $(5\Delta x/6 + \Delta x/2)(1)$
nodal area normal to y axis is $(\Delta y).(1)$

for point 6:

$$\frac{11/12\,\Delta x}{\Delta y}(T_{s1} - T_6) + \frac{11/12\,\Delta x}{\Delta y}(T_{13} - T_6) + \frac{\Delta y}{5/6\,\Delta x}(T_7 - T_6) + \frac{\Delta y}{\Delta x}(T_5 - T_6) = 0$$

$$11/10\, T_{s1} - 3.2\, T_6 - 5/6\, T_6 + 11/10\, T_{13} + T_7 + 5/6\, T_5 = 0$$

$$1.1\, T_{s1} - 121/30\, T_6 + 1.1\, T_{13} + T_7 + 5/6\, T_5 = 0$$

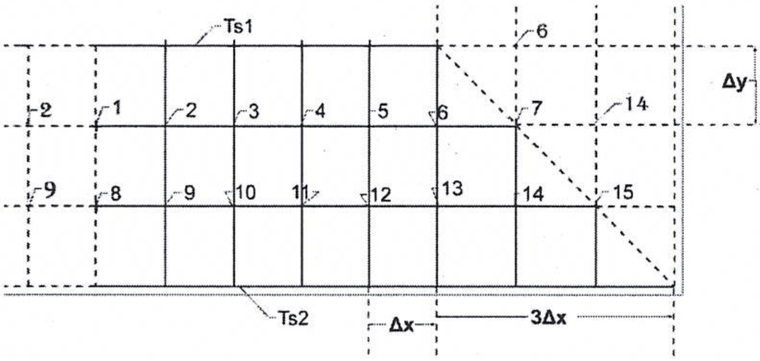
for point 13;

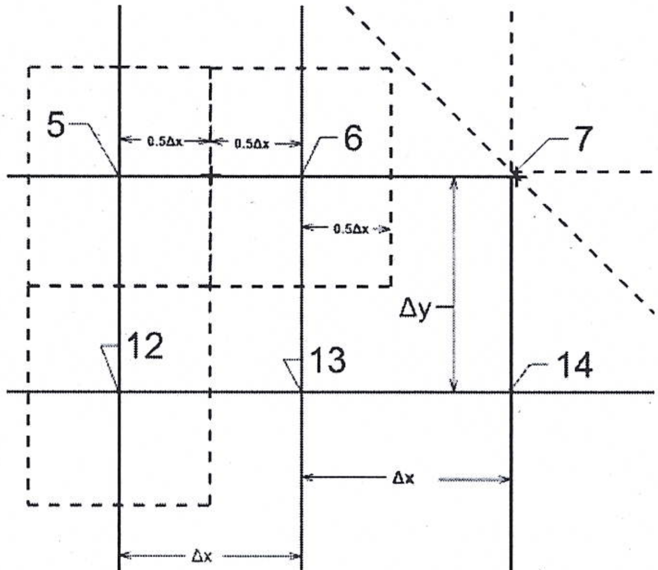
$$\frac{11/12\,\Delta x}{\Delta y}(T_6 - T_{13}) + \frac{11/12\,\Delta x}{\Delta y}(T_{s2} - T_{13}) + \frac{\Delta y}{5/6\,\Delta x}(T_{14} - T_{13}) + \frac{\Delta y}{\Delta x}(T_{12} - T_{13}) = 0$$

$$1.1\, T_6 - 121/30\, T_{13} + 1.1\, T_{s2} + T_{14} + 5/6\, T_{12} = 0$$

SCALE 1:6

$$\frac{\Delta x}{\Delta y} = 1$$





for all scales all energy balance equations for points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15 are identical respectively.

for points 6 and 13 nodal area normal to x axis is $(\Delta x)(1)$
nodal area normal to y axis is $(\Delta y).(1)$

for point 6;

$$\frac{\Delta x}{\Delta y}(T_{s_1} - T_6) + \frac{\Delta x}{\Delta y}(T_{13} - T_6) + \frac{\Delta y}{\Delta x}(T_7 - T_6) + \frac{\Delta y}{\Delta x}(T_5 - T_6) = 0$$

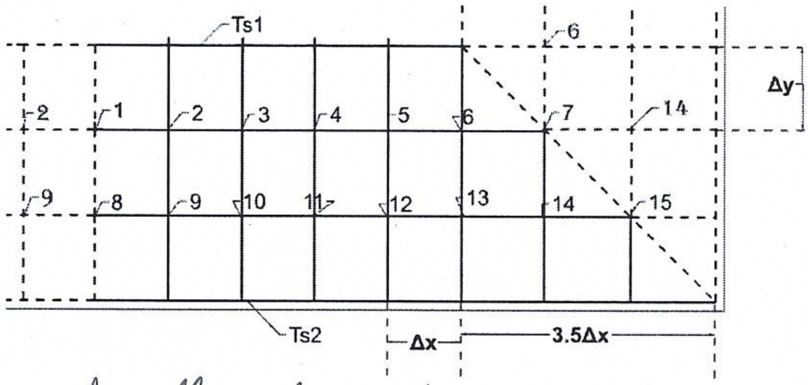
$$T_{s_1} + T_{13} + T_7 + T_5 - 4T_6 = 0$$

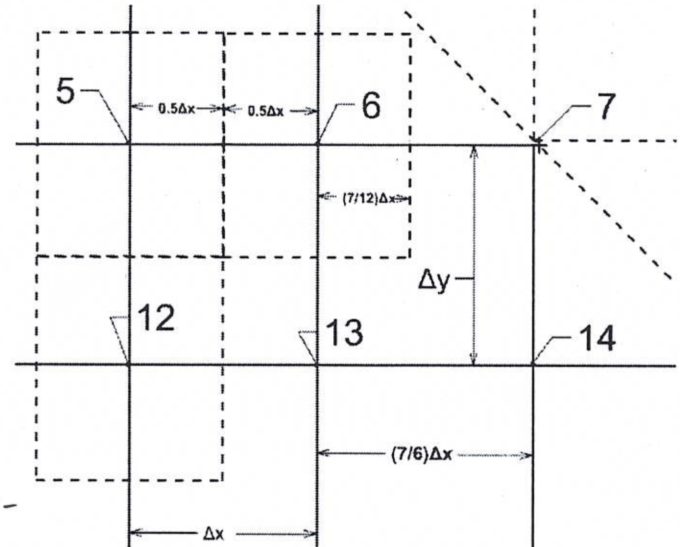
for point 13;

$$T_{12} + T_{14} + T_6 + T_{s_2} - 4T_{13} = 0$$

SCALE 1:7

$$\frac{\Delta x}{\Delta y} = \frac{6}{7}$$





for all scales all energy balance equations for points 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15 are identical respectively.

for points 6 and 13 nodal area normal to x axis is $(7/12\,\Delta x + \Delta x/2)(1)$

nodal area normal to y axis is $(\Delta y)(1)$

for point 6;

$$\frac{10/6\,\Delta x}{\Delta y}(T_{s1} - T_6) + \frac{10/6\,\Delta x}{\Delta y}(T_{13} - T_6) + \frac{\Delta y}{7/6\,\Delta x}(T_7 - T_6) + \frac{\Delta y}{\Delta x}(T_5 - T_6) = 0$$

$$10/7\, T_{s1} - 211/42\, T_6 + 10/7\, T_{13} + T_7 + 7/6\, T_5 = 0$$

for point 13;

$$\frac{10/6\,\Delta x}{\Delta y}(T_6 - T_{13}) + \frac{10/6\,\Delta x}{\Delta y}(T_{s2} - T_{13}) + \frac{\Delta y}{7/6\,\Delta x}(T_{14} - T_{13}) + \frac{\Delta y}{\Delta x}(T_{12} - T_{13}) = 0$$

$$10/7\, T_6 - 211/42\, T_{13} + 10/7\, T_{s2} + T_{14} + 7/6\, T_{12} = 0$$

By writing a MATLAB code, we found the temperature of each point with the matrix solution. URL for MATLAB file and solutions given below

.m file:

https://drive.google.com/drive/folders/14w0tjpptJCgzrokca8mnvzlpPs5kxyKi?usp=drive_link

Result for A/a=1.1 :

T1 = 73.07, T2 = 72.94, T3 = 72.40, T4 = 70.88, T5 = 66.61, T6 = 54.29, T7 = 42.34,

T8 = 46.41, T9 = 46.29, T10 = 45.79, T11 = 44.49, T12 = 41.28, T13 = 34.03,

T14 = 30.39, T15 = 25.20

S1 = 14.35

Result for A/a=1.2:

T1 = 73.22, T2 = 73.16, T3 = 72.92, T4 = 72.26, T5 = 70.42, T6 = 65.23, T7 = 49.83,

T8 = 46.55, T9 = 46.50, T10 = 46.28, T11 = 45.68, T12 = 44.20, T13 = 40.70, T14 = 34.44,

T15 = 27.22

S2 = 6.91

Result for A/a=1.3:

T1 = 73.04, T2 = 72.89, T3 = 72.29, T4 = 70.57, T5 = 65.74, T6 = 51.64, T7 = 40.62,

T8 = 46.38, T9 = 46.24, T10 = 45.70, T11 = 44.26, T12 = 40.75, T13 = 33.01, T14 = 29.61,

T15 = 24.81

S3 = 4.5

Result for A/a=1.5:

T1 = 73.22, T2 = 73.17, T3 = 72.94, T4 = 72.29, T5 = 70.51, T6 = 65.43, T7 = 50.00,

T8 = 46.56, T9 = 46.50, T10 = 46.29, T11 = 45.73, T12 = 44.32, T13 = 41.03, T14 = 34.58,

T15 = 27.29

S4 = 2.54

Result for A/a=1.6:

T1 = 73.22, T2 = 73.17, T3 = 72.94, T4 = 72.29, T5 = 70.50, T6 = 65.39, T7 = 49.99,

T8 = 46.56, T9 = 46.50, T10 = 46.29, T11 = 45.73, T12 = 44.33, T13 = 41.08, T14 = 34.59,

T15 = 27.30

S5 = 2.16

Result for A/a=1.7:

T1 = 73.25, T2 = 73.20, T3 = 73.02, T4 = 72.52, T5 = 71.12, T6 = 67.15, T7 = 51.20,

T8 = 46.58, T9 = 46.54, T10 = 46.37, T11 = 45.93, T12 = 44.81, T13 = 42.21, T14 = 35.26,

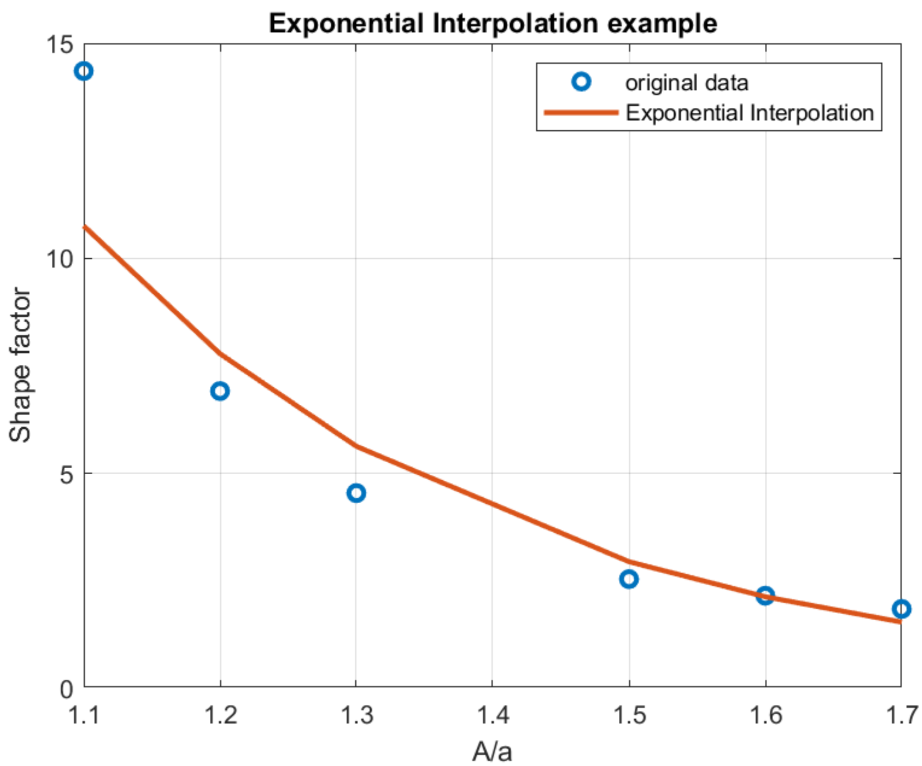
T15 = 27.63

S6 = 1.85

We used curve fitting methods and obtain different equations for different fitting techniques. These equations and their graphs given below. You need to know that the continuous equations and functions given below, obtained by assuming without knowledge of nature of shape factor is changing after scale approximately 1.4 on the other hand discrete, more convenient, equation and the graph of it given after continuous ones.

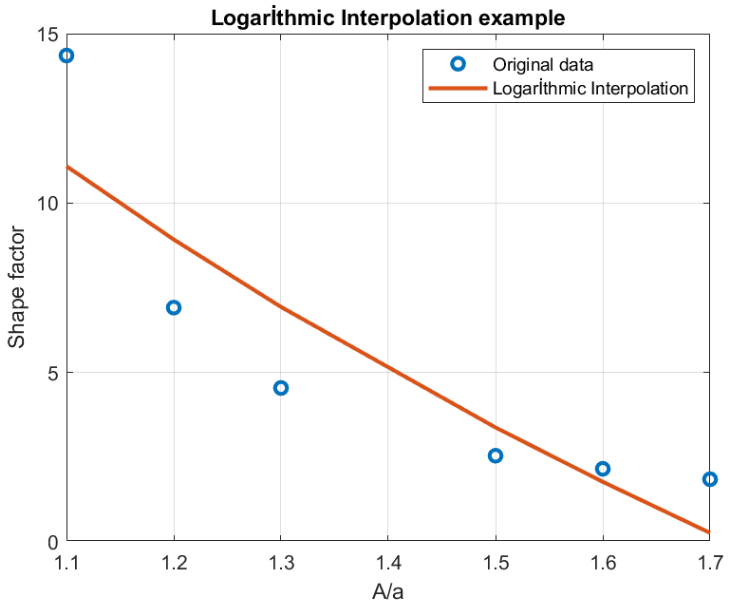
Exponential Function:

$S = 377.9841 * e^{(-3.2359 * A/a)}$



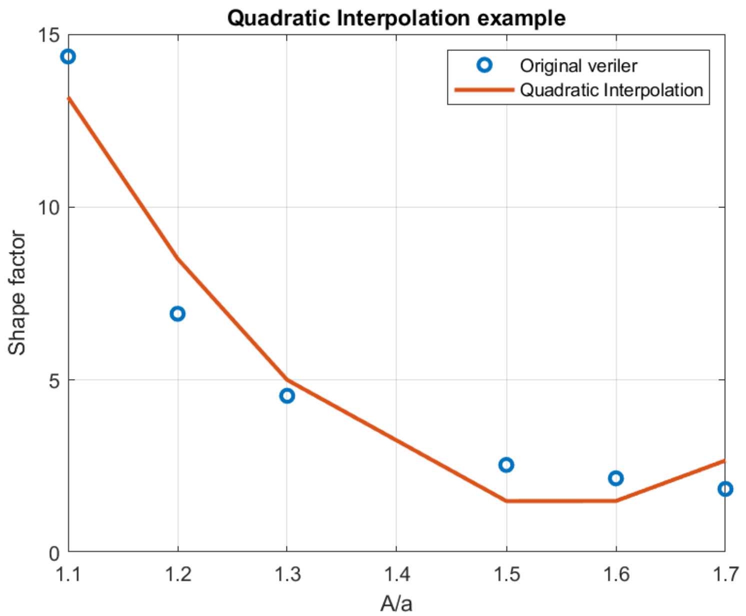
LogarÌthmic Function:

S = -24.8598 * log(A/a) + 13.4566



Quadratic Function:

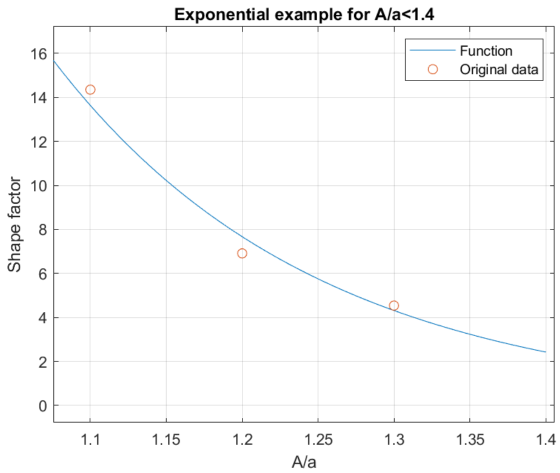
S = 58.4406 * (A/a)^2 + -181.1468 * (A/a) + 141.7281

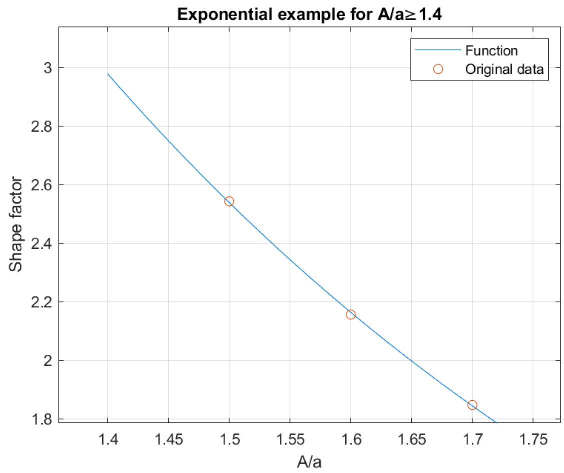


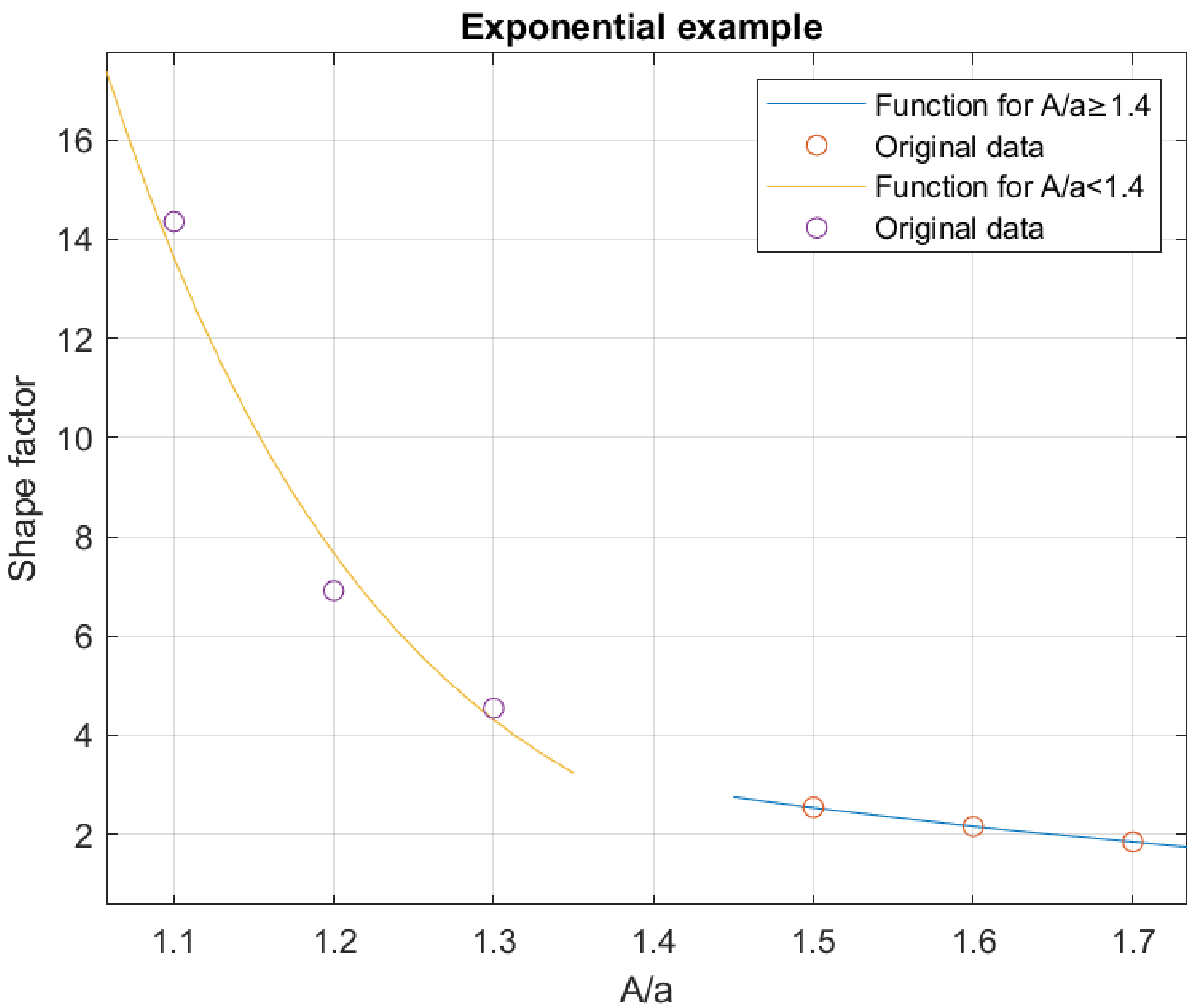
Descrete Exponential Function:

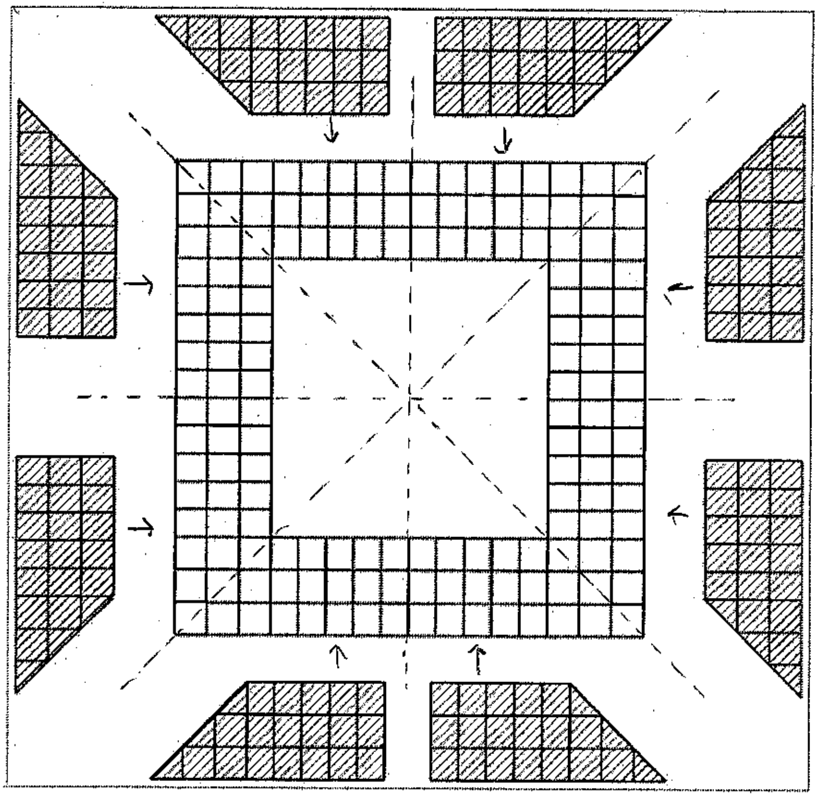
$$S = 7629.2523 * e^{(-5.7522 * A/a)} \quad ; \quad A/a < 1.4$$

$$S = 27.8536 * e^{(-1.5969 * A/a)} \quad ; \quad A/a \geq 1.4$$



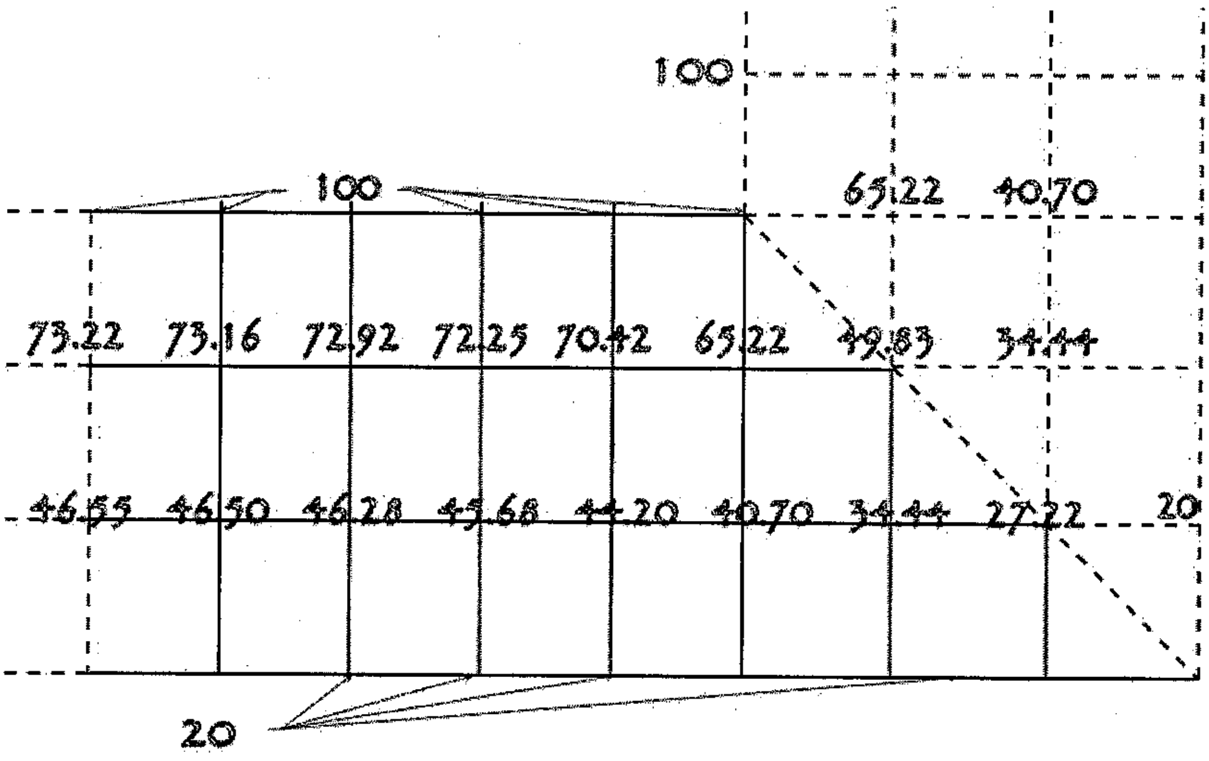






Since no heat can across a symmetry lines, the "mirror effect" occurs. Thus temperature distribution of one eighth of shape of any given A/a scale could be mirrored about symmetry axises as shown left.

Temperature distribution of one eighth of shape for 1.2 scale specified as $T_{s_1} = 100°C$ and $T_{s_2} = 20°C$ is given below.



Temperature distribution of one eighth of shape for 1.5 scale specified as $T_{s_1} = 100°C$ and $T_{s_2} = 20°C$ is given below.

It is important to notice that at corners, horizontal dimensions are different than side wall dimensions although its seems same.